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RELATIONS FOR GASDYNAMIC DISCONTINUITIES FOR TWO-PHASE FLOWS OF A NONEQUILIBRIUM CONDENSING VAPOR

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Equations of characteristics and relations along characteristics are presented along with relations for normal and oblique shock waves for a two-phase flow of a nonequilibrium condensing vapor.

1. We will examine a two-dimensional steady-state flow of a nonequilibrium condensing vapor in the initial condensation zone in a one-velocity approximation. We assume the vapor phase to be a perfect gas.

The system of conservation equations in this case has the form [1]

$$\operatorname{div}(\rho\vec{w}) = 0,\ \rho(\vec{w}\cdot\nabla)\vec{w} + \nabla p = 0,\ \operatorname{div}(\rho I\vec{w}) = 0, \tag{1}$$

where

$$\rho = \rho' s + \rho''(1-s);\ I = i + \frac{w^2}{2};\ i = \frac{\rho' s i' + \rho''(1-s) i''}{\rho}; \tag{2}$$

$$p = \rho'' R T'';\ i'' = \frac{k}{k-1}\frac{p}{\rho''}. \tag{3}$$

Excluding the derivatives of density from system (1) with the use of equation of state (3), we obtain

$$(u^2 - a_{tp}^2)\frac{\partial u}{\partial x} + uv\frac{\partial u}{\partial y} + uv\frac{\partial v}{\partial x} + (v^2 - a_{tp}^2)\frac{\partial v}{\partial y} = m a_{tp}^2, \tag{4}$$

where a_{tp} is the analog of the speed of sound in the two-phase medium,

$$a_{tp} = \sqrt{\frac{k-1}{1-ks}},\ i = \sqrt{\frac{k-1}{1-ks}\ \frac{\rho' s i' + \rho''(1-s) i''}{\rho}}; \tag{5}$$

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TABLE 1. Coefficients of Formulas to Calculate Shock Waves in a Two-Phase Flow

Coefficient	$\hat{y}$=const	s = const
α_1	$\frac{(k+1)s_1-2ks_*}{(k+1)(1-s_1)} - \frac{2\rho' s_1 i'}{\rho_1 a_*^2}\alpha_4$	$\frac{2k(s_1-s_*)}{k+1-2ks_1}$
α_2	$\frac{\beta_3 s_1}{(k+1)(1-s_1)}$	0
α_3	$1+\alpha_1+\alpha_2\lambda_1^2$	λ_1^2
α_4	$\frac{k-1}{(k+1)(1-s_1)}$	$\frac{k-1}{k+1-2ks_1}$
β_1	$k+1$	$k+1-2ks_1$
β_2	0	$2\rho' s_1 i'(k-1)$
β_3	$k-1+2ks_1$	$k-1$
γ_1	$\alpha_2+\alpha_4$	$\frac{\beta_3}{\beta_1}$
γ_2	$\frac{2(1-ks_1)}{(k+1)(1-s_1)}(1-\gamma_4)$	$\frac{2(1-ks_1)}{\beta_1}$
γ_3	$-\alpha_4\gamma_4$	0
γ_4	$\frac{2\rho' s_1 i'}{\rho_1 a_*^2}\cdot\frac{k-1}{k+1-2ks_*}$	—
γ_5	0	β_2
γ_6	$\left[1-\gamma_4-\frac{\gamma_4(k-1)M_1^2}{2(1-ks_1)}\right](k+1)$	β_1
γ_7	$\gamma_6\frac{k-1}{k+1}$	β_3
γ_8	0	γ_5
γ_9	0	$\frac{\gamma_5}{2(1-ks_1)}$

$$m=\frac{\rho' i'-\rho'' i''}{\rho i}\left(u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}\right)+\frac{\rho' s}{\rho i}\left(u\frac{\partial i'}{\partial x}+v\frac{\partial i'}{\partial y}\right). \quad (6)$$

Comparing the quantity a_{tp} with the adiabatic speed of sound in a perfect gas $a_{one} = \sqrt{kp/\rho''}$, we obtain the following equation connecting them:

$$a_{tp}=\frac{a_{one}}{\sqrt{1-ks}}\sqrt{1-\frac{\rho' s}{\rho}\left(1-\frac{i'}{i''}\right)}. \quad (7)$$

2. Equation (4) can be used together with the equations of motion to find characteristics of the flow. Having performed familiar transformations (see [2], for example), we find that the characteristics of the two-phase flow being examined are lines determined by the equations dy/dx = v/u, as well as two families of characteristic curves satisfying the equality:

$$\frac{dy}{dx}=\frac{uv\pm a_{tp}\sqrt{w^2-a_{tp}^2}}{u^2-a_{tp}^2}=\frac{\zeta\sqrt{M^2-1}\pm 1}{\sqrt{M^2-1}\mp\zeta}, \quad (8)$$

where

$$\zeta=v/u;\ M=w/a_{tp}. \quad (9)$$

The compatibility conditions used along characteristics (8) have the form

$$d\zeta\pm\frac{1+\zeta^2}{\rho w^2}\sqrt{M^2-1}\,dp\pm\frac{(1+\zeta^2)^{\frac{3}{2}}}{w(\sqrt{M^2-1}\mp\zeta)}m\,dx=0. \quad (10)$$

It is apparent that in the absence of moisture (s = 0, m = 0) Eq. (10) coincides with the well-known relation from single-phase gasdynamics [2].

3. The problem of calculating a shock wave in a two-phase flow was examined in [3, 4] with different assumptions. The study [3] only addressed the problem of transition across the shock for the process of conservation of thermodynamic equilibrium of the phases and long-term relaxation. The investigation [4] proposed the iteration method for the first of these cases.

Let the following parameters be known before the shock: w_1, p_1, ρ''_1, ρ'_1, s_1, $T''_1(i''_1)$ $T'_1(i'_1)$. We need to find the values of the following parameters after the shock: w_2, p_2, ρ''_2, ρ'_2, s_2, $T''_2(i''_2)$, $T'_2(i'_2)$.

Applying the laws of conservation to the mass of a moist vapor enclosed in an elemental cylinder with its cross section normal to the direction of the incoming flow and equal to unity, we obtain the following equations:

$$\rho_1 w_1 = \rho_2 w_2,\ p_2 = p_1 + \rho_1 w_1 (w_1 - w_2),\ 2i_2 + w_2^2 = 2i_1 + w_1^2 . \tag{11}$$

To close this system we need additional conditions pertaining to the change in the parameters of the liquid phase in the transition across the shock. We will assume that the temperature of the moisture remains constant and we will examine two cases:

a) The mass concentration $\hat{y}$ of the liquid phase is constant ($\hat{y}$ = const). This corresponds to a process with a long relaxation period, in which phase transitions begin to develop only behind the shock front.

b) The volume concentration s of the liquid phase is constant (s = const). In this case, the process occurs in such a way that some of the moisture is evaporated as a result of the transition across the shock.

Having used familiar transformations and excluded density ρ_2 and pressure p_2 from these equations, we obtain

$$w_1 w_2 = a_*^2 + \alpha_1 a_*^2 + \alpha_2 w_1^2 , \tag{12}$$

where α_1 and α_2 are coefficients shown in Table 1. Also shown are the coefficients of the following formulas.

Introducing the velocity coefficient $\lambda = w/a_*$, we reduce Eq. (12) to the form

$$\lambda_1 \lambda_2 = 1 + \alpha_1 + \alpha_2 \lambda_1^2. \tag{13}$$

It is clear that in a single-phase flow with s = 0 Eq. (13) becomes the familiar Prandtl equation for a normal shock [5].

The rest of the parameters behind the normal shock can be found from the formulas

$$\frac{\rho_2}{\rho_1} = \frac{\lambda_1^2}{1 + \alpha_1 + \alpha_2 \lambda_1^2},\quad \frac{p_2 - p_1}{\rho_1 w_1^2} = 1 - \frac{1 + \alpha_1 + \alpha_2 \lambda_1^2}{\lambda_1^2},$$

$$s_2 = \frac{\lambda_1^2}{\alpha_3} s_1,\quad \rho''_2 = \frac{\rho_2 - \rho' s_2}{1 - s_2},\quad T''_2 = \frac{p_2}{\rho''_2 R}.$$

4. In the equation $2i + w^2$ = const, valid for steady-state adiabatic two-phase flow, we can find the constant from the condition $a_{tp} = a_*$ with $w = a_*$. From this, using (5) we obtain the equation

$$\frac{2(1-ks)}{k-1}\frac{a_{tp}^2}{w^2} + 1 = \frac{k+1-2ks_*}{k-1}\frac{a_*^2}{w^2},$$

from which we find the connection between λ and M for a two-phase flow:

$$\lambda = \sqrt{\frac{k+1-2ks_*}{2(1-ks)}}\ \frac{M}{\sqrt{1 + \dfrac{k-1}{2(1-ks)} M^2}} .$$

Instead of the number $M = w/a_{tp}$ we sometimes examine the number $M'' = w/a_{one}$ for the vapor phase. Using Eq. (7), we have

$$M'' = \sqrt{\frac{1 - \dfrac{\rho' s}{\rho}\left(1 - \dfrac{i'}{i''}\right)}{1 - ks}}\, M.$$

5. Equations (11) can be used to find the relationship between the pressure p_2 and the density ρ_2 of the two-phase flow behind the shock and the analogous quantities p_1 and ρ_1 ahead of the shock:

$$\frac{\rho_2}{\rho_1} = \frac{\beta_1 p_2 + (k-1) p_1 + \beta_2}{\beta_3 p_2 + (k+1-2ks_1) p_1 + \beta_2}. \qquad (14)$$

These formulas are the two-phase analog of the Hugoniot curve for a single-phase flow.

6. Let us examine an oblique shock wave. Applying the laws of conservation to the mass of moist vapor enclosed within an elementary cylinder with its generatrix coincident with the direction of a normal to an element of the surface of a severe shock and reasoning similarly to the case of a normal shock, we arrive at the following relations:

$$\begin{gathered} \rho_1 w_{1n} = \rho_2 w_{2n}, \; w_{1\tau} = w_{2\tau}, \\ p_2 = p_1 + \rho_1 w_{1n}(w_{1n} - w_{2n}), \\ 2i_2 + w_{2n}^2 = 2i_1 + w_{1n}^2, \end{gathered} \qquad (15)$$

from which, similarly to (12), we obtain

$$w_{1n} w_{2n} = a_*^2 + \alpha_1 a_*^2 + \alpha_2 w_{1n}^2 - \alpha_4 w_{1\tau}^2. \qquad (16)$$

Having chosen the coordinate axes so that the x axis coincides with the direction of the velocity $\vec{w}_1$ to the surface of the discontinuity and having used φ to designate the angle of inclination of the tangent at the given point of the surface to the x axis and δ to designate the angle of inclination of the velocity vector $\vec{w}_2$ to the x axis, we transform the equations of system (15) so that we obtain

$$\begin{gathered} w_2 = w_1 \frac{\cos\varphi}{\cos(\varphi - \delta)}, \\ \rho_2 = \rho_1 \frac{\operatorname{tg}\varphi}{\operatorname{tg}(\varphi - \delta)}, \\ p_2 = p_1 + \rho_1 w_1^2 \frac{\sin\varphi \sin\delta}{\cos(\varphi - \delta)}. \end{gathered} \qquad (17)$$

Using Eqs. (17) and (16), we obtain the following equation:

$$\operatorname{tg}\delta = \operatorname{ctg}\varphi \frac{(1-\gamma_1)\sin^2\varphi - \frac{\gamma_2}{M_1^2} - \gamma_3}{1 - (1-\gamma_1)\sin^2\varphi + \frac{\gamma_2}{M_1^2} + \gamma_3}. \qquad (18)$$

Equation (18) makes it possible to determine the angle φ for a prescribed direction δ behind an oblique shock. Knowing φ and δ, we can use Eqs. (17) to find the parameters w_2, ρ_2, and p_2 and then determine the remaining quantities behind the shock.

If, besides the number M_1, we also know the pressure ratio p_2/p_1, then, by simultaneously solving Eqs. (14), (17), and (18), we can find the angle of inclination of the surface of the discontinuity:

$$\operatorname{ctg}\varphi = \left[\frac{[2k(1-s_1)p_1 + \gamma_5] M_1^2}{\gamma_6 p_2 + \gamma_7 p_1 + \gamma_8} - 1\right]^{\frac{1}{2}}. \qquad (19)$$

Having inserted (19) into (18), we obtain a formula for the increment in the angle of inclination of velocity in the transition across an oblique shock:

$$\operatorname{tg}\delta = \frac{p_2 - p_1}{\left[\frac{k(1-s_1)p_1}{1-ks_1} + \gamma_9\right] M_1^2 - p_2 + p_1} \left\{\frac{[2k(1-s_1)p_1 + \gamma_5] M_1^2}{\gamma_6 p_2 + \gamma_7 p_1 + \gamma_8} - 1\right\}^{\frac{1}{2}}.$$

It is apparent that in the absence of moisture (s = 0) the formulas obtained here for shock waves agree with the well-known relations of single-phase gasdynamics [5]. The parameters of a two-phase flow behind a shock that are found with these formulas lead to significant differences from the single-phase analogs. For example, with a mass moisture content of 5-8%, these deviations reach 6-10%.

7. The relations obtained in this article can be used directly to calculate shock waves and rarefaction waves in two-phase media. They can also be used in different applications of numerical through-count algorithms to problems of calculating nonequilibrium two-phase flows. It is known [6, 7] that methods such as a steady-state analog of the Godunov difference scheme for calculating supersonic flows include solutions of gasdynamic problems on the breakdown of a shock and different analogs of the latter. Here, the computing algorithms contain formulas connecting parameters at the shock and in the rarefaction wave. Thus, the relations obtained in this article for discontinuities of gasdynamic quantities can be used directly in devising methods of calculating steady-state two-phase flows based on the above-noted numerical schemes.

NOTATION

a_{tp}, speed of sound in two-phase medium; a_{one}, speed of sound in single-phase medium; i, enthalpy; I, total enthalpy; k, exponent of adiabatic curve; M, Mach number; p, pressure; R, gas constant; s, volume concentration of liquid phase; T, temperature; u, v, velocity components; w, velocity; y, mass concentration of liquid phase; δ, angle of rotation of flow in transition across an oblique shock; φ, angle of inclination of surface of a severe shock; λ, velocity coefficient; ρ, density. Indices: n, τ, directions of normal and tangent to an element of the shock surface; 1 and 2, parameters before and after shock; ', liquid phase; ", vapor phase; *, parameters in the critical section.

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